Mathematics II

029

25/07/2023   08.30 AM -11.30 AM

NESA NATIONAL EXAMINATION AND
SCHOOL INSPECTION
AUTHORITY

ADVANCED LEVEL NATIONAL EXAMINATIONS, 2022-2023

SUBJECT: MATHEMATICS II

COMBINATIONS:

- MATHEMATICS-CHEMISTRY-BIOLOGY **(MCB)**
- MATHEMATICS -COMPUTER SCIENCE-ECONOMICS **(MCE)**
- MATHEMATICS-ECONOMICS-GEOGRAPHY **(MEG)**
- MATHEMATICS -PHYSICS-COMPUTER SCIENCE **(MPC)**
- MATHEMATICS-PHYSICS-GEOGRAPHY **(MPG)**
- PHYSICS-CHEMISTRY-MATHEMATICS **(PCM)**

DURATION: 3 HOURS

INSTRUCTIONS:

1) Write your names and index number on the answer booklet as written on your registration form, and **DO NOT** write your names and index number on additional answer sheets if provided.

2) Do not open this question paper until you are told to do so.

3) This paper consists of **two** sections: **A** and **B**.

   Section A: Attempt **all** questions.   **(55 marks)**

   Section B: Attempt **only three** questions.   **(45 marks)**

4) **Geometrical instruments and silent non-programmable calculators may be used.**

5) Use only a **blue** or **black** pen.

SECTION A: ATTEMPT ALL QUESTIONS (55 marks)

1) State the cosine and sine laws which are used to solve practical problems involving triangles and angles. **(4 marks)**

2) In a geometric progression, insert 4 geometric terms that are between 2 and 6250. **(4 marks)**

3) Solve $e^{x-1} - 18e^{1-x} - 3 = 0$ **(4 marks)**

4) Find the differential equation of all straight lines passing through the origin. **(2 marks)**

5) Find the vector, parametric and symmetric equations of the line (l) passing through the point $A(3,-2,4)$ with direction vector $\vec{u} = (2,3,5)$. **(4 marks)**

6) Find square roots of the complex number 3+4i **(3 marks)**

7) From the top of a cliff, 100 m above sea level, the angle of depression to a ship sailing past is 17 degrees. How far is the ship from the base of the cliff to the nearest meter? **(3 marks)**

8) Use Gauss – Jordan method of elimination to solve:

$$\begin{cases} -3x - 2y + 4z = 9 \\ \quad\; 3y - 2z = 5 \\ 4x - 3y + 2z = 7 \end{cases}$$

(4 marks)

9) Write the 3 terms of the Maclaurin expansion of $f(x) = \ln(1 + e^x)$ **(3 marks)**

10) Let **A** be a 2×2 matrix with $tr(A) = 6$ and $det(A) = 5$.
Find the eigenvalues of A. **(3 marks)**

11) Let $t: \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation such that
$t(1,0,0) = (2,4,-1)$, $t(0,1,0) = (1,3,-2)$ and $t(0,0,1) = (1,-2,2)$.
Find $t(0,3,-1)$. **(3 marks)**

12) Verify if $T = E^2 \to E^2$ defined by $T((x_1, x_2)) = (x_1 + x_2, \; x_1 - x_2 + 1)$
is linear or not linear. **(3 marks)**

13) Suppose that you are observing the behavior of cell duplication in a laboratory. If, in one of the experiments, you started with 1,000,000 cells and the cell population decreased by ten percent every minute.

    a) Write an equation with base (0.9) to determine the number of cells after **t** minutes. **(1 mark)**

    b) Determine how long it would take the population to reach a size of 10 cells. **(3 marks)**

$$y = 2x + 1$$

14) Given that: $y = 3x + 1$

$$x = 4$$

    a) Find the coordinate points of intersection of the lines.   **(3 marks)**

    b) Sketch a graph of the given lines in the same

        two dimensions.   **(2 marks)**

    c) Find the area of the region found in (b).   **(2 marks)**

15) a) Find the constant **c** such that the function :

$$f(x) = \begin{cases} cx^2 & 0 < x < 3 \\ 0 & otherwise \end{cases}$$

        is a density function   **(2 marks)**

    b) Compute $P(1 < x < 2)$ **(2 marks)**

SECTION B: ATTEMPT THREE QUESTIONS ONLY (45 marks)

16) Give that $f(x) = \frac{1}{2}x^2 e^{x+1}$

    a) Find the domain of $f(x)$,   **(1 marks)**

    b) Find relative asymptotes (if any),   **(4 marks)**

    c) Study the first and second derivative with variation table,   **(8 marks)**

    d) Sketch the curve of $f(x)$.   **(2 marks)**

17) Given the differential equation

$$\frac{d^2y}{dx^2} - 2k\frac{dy}{dx} + k^2 y = 12xe^{kx}, k > 0$$

    a) Find a general solution of differential equation given that
    $y = Px^3 e^{kx}$ where **P** is a constant and part of the solution.   **(11 marks)**

    b) Given further that $y = 1, \frac{dy}{dx} = 0$ at $x = 0$ show that

$$y = e^{kx}(2x^3 - kx + 1)$$

    (4 marks)

18) The table below shows the marks scored by 10 students in Biology and Chemistry test.

Biology (x)	8	7	6	9	8	9	7	8	5	6
Chemistry (y)	7	8	7	9	8	8	7	9	7	5

a) Find Mean, **(5 marks)** **(1 mark)**
b) Find Variance, **(1 mark)**
c) Find Standard deviation of x and y **(1 mark)**
d) Find covariance of x and y **(1 mark)**
e) Find correlation coefficient **r** and interpret it **(2 marks)**
f) Find an equation of a line that best fits
in the form of $y = a + bx$ **(3 marks)**
g) If a student scored 7.5 in Biology, predict his/her
score in Chemistry. **(2 marks)**

19) For the conic defined by $9x^2 - 16y^2 - 18x - 64y - 199 = 0$

a) Write the given conic in a standard form. **(2 marks)**
b) Name the conic represented in (a). **(1 mark)**
c) Find the:

   i) Coordinates of the centre. **(2 marks)**
   ii) Vertices. **(2 marks)**
   iii) Foci. **(4 marks)**
   iv) Equations of the asymptotes then sketch the graph. **(4 marks)**

20) It is given that $Z = \dfrac{1 + 8i}{1 - 2i}$

a) Express Z in the form $x + iy$, where x and y are real numbers. **(4 marks)**
b) Find the modulus and argument of Z. **(4 marks)**
c) Show clearly that: $\arctan 8 + \arctan 2 + \arctan\frac{2}{3} = \pi$ **(7 marks)**

-END-

CamScanner